

Two effective temperatures in traffic flow models: analogies with granular flow

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Abstract

We present a model of traffic flow, with rules that describe the behaviour of *automated vehicles* in an *open system*. We show first of all that the fundamental diagram of this system collapses to a point, where states of free and jammed traffic can coexist in phase space. This leads us to consider separately the roles of average velocities and densities as better descriptors of the actual state of the traffic. Next, we observe that the transition between free and jammed traffic as a function of the *braking parameter* R is different for high and low initial densities, in the steady state; it turns out to be 'smeared out' for low densities, a behaviour which is already portended by the transient behaviour of the system. Our results indicate strongly that, at least for such models, two effective temperatures (one related to R , and the other to the density) are needed to describe the global behaviour of this system in statistical mechanical terms. Analogies with granular flow are discussed in this context.

1 Introduction

Traffic flow[1] is a subject of interdisciplinary[2] interest at the present time, both to do with the very real problems of congestion on busy highways[3], as well as an example of a complex system whose behaviour has yet to be fully understood. The study of this system started as early as 1934 by Greenshields

with a study on traffic capacity[4]. Since its inception, the study of traffic flow was based on stochastic processes[5]. In 1955 Lighthill and Witham described the existence of density waves as well as shock waves[6] in a continuous model of traffic flow. This was also obtained as a limiting case from a Boltzmann-like model by Prigogine and co-workers[7]. In these models, traffic flow was treated in an average sense. The discussion on the "temperature" of this kind of systems has also been discussed from the thermodynamic point of view[8]. On the other other hand, car-following theories deal with equations of motion for individual cars, by using the analogues of Newton's equations of motion[2]. As early as 1958, Chandler et al. [9] have argued that one should take into account the reaction times of drivers, in response to the cars in front of them. From this "microscopic" perspective one obtains nonlinear differential equations which describe flow-density relations characteristic of current models [10]. In 1956 Gerlough proposed a cellular automaton model for traffic flow; similar models have been the subject of recent work[11]. Finally, analogies between vehicular traffic with granular flow, while longstanding, [12] have also been the subject of much current interest[13],[15],[14]. In this paper we concern ourselves largely with these two latter aspects.

In this paper, we focus on a new and rather specific analogy between traffic and granular flow [15] which concerns the role of temperature; the results of our model suggest that *two* effective temperatures are needed to describe the flow of traffic in thermodynamic terms, which is analogous to the situation that obtains in models of shaken sand [16].

We have modified[17] the extensively studied model of Nagel and Schreckenberg (NaSch)[2],[11], [18] to consider *automated vehicles*. This was motivated by a realistic system, which is a car equipped with an infrared sensor to determine distance and velocity from its neighbouring vehicles: this was recently used by the Intelligent Transport Society to conduct studies on automated vehicles [3]. The objective of our model is also to study the effect of open boundary conditions (as occurs in reality) on vehicular traffic. In our specific case, this involves four cell-updating steps involving braking, stochastic driver reaction, and car movement/acceleration. The chief modification we have made to the NaSch model involves stochastic changes to the car occurring *before* the braking step, to model the behaviour of an automated car or an 'anticipatory' driver.

In open systems, nonequilibrium conditions greatly modify the underlying physics; even in the steady state, we find that the construction of the phase diagram is totally different, involving as it does a collapse of the phase space, relative to the so-called 'fundamental' diagram obtained for the closed version[19], [11]. This necessitates a description of the traffic in terms of densities and velocities, since the flux is no longer a good variable for this open system.

Our results indicate that for high values of the initial density, there is a sharp transition between free and jammed flow at a certain value R_c of a parameter R (see below) which we call the 'braking parameter' of the system. This transition gets smeared out, in the case of lower densities, so that it is possible to get free flow *even* at values of $R \gg R_c$. We find that both R and the density are needed to describe the behaviour of the system in thermodynamic terms, just as in the

case of shaken powders. In terms of the analogy with granular flow [20], the parameter R is found to be related (inversely) to the shaking intensity, and is like a 'fast dynamics' temperature, while the density is like a 'slow dynamics' temperature, much like the Edwards' compactivity [21] for granular systems. We will give an explanation of this in terms of individual and collective effects below, as has already been done in the case of granular flow [22]. This is borne out by the study of velocity correlation functions, at specific values of R , which manifest a 'dynamical clustering' as observed in models of vibrated sandpiles [23].

This paper is organized as follows. In the first section we present our model. In the next section we present results concerning both transient and steady-state regimes in the open systems under study; we also include a comparison with a system with periodic boundary conditions. Lastly, we summarise our observations, and compare our predictions with observations on real traffic.

2 The model

Most studies involving cellular automata modelling of traffic have sought to focus on its evolution in closed systems, subject to periodic boundary conditions. However, for both practical and theoretical reasons, open boundary conditions are preferable. As we all know, traffic flow in the real world always occurs in open systems, i.e. those where cars are interchanged between some local environment and its surroundings; thus for example, the number of cars is *not* conserved in general in any section of a highway. In particular a one-dimensional example of an open system could be, for example, a situation where a multilane road is reduced to one lane, e.g. due to road construction. Recently, a study of open boundary conditions has been performed characterizing the phase transition from free to jammed flow in terms of the input and output rates of cars [24]. What we seek to do in this study is to extend this line of research, and model in particular the specific features which obtain when automated vehicles are considered.

We base our model on the NaSch [11] cellular automaton model; our most important modification involves changing the order of the operators and an amendment made to the braking step. Our model is defined as follows:

Consider a one-dimensional array of L cells. Each cell can either be empty or occupied by one car with velocity $v \in \{v_{\min}, \dots, v_{\max}\}$ (with $v_{\min} = 1$ and $v_{\max} = 5$). The state of car j ($j = 1, \dots, N$) is characterized by an internal parameter v_j ($v_j = 1, \dots, v_{\max}$), the instantaneous velocity of the vehicle. Let x_j denote the position of the j th vehicle; the distance between cars j and $j+1$ is then given by $d_j = x_{j+1} - x_j$. We propose dynamical rules involving three quantities: the velocity v , the braking parameter R and the intercar distance d_j . In order to obtain the state of the system at time $t+1$ (which we denote by primed quantities) from the state at time t , the following four rules are applied to all cars:

1. Acceleration (\mathcal{A}): Each vehicle increases its current velocity if $v_j < v_{\max}$:

$$v'_j \leftarrow \min(v_j + 1, v_{\max}) \quad (1)$$

2. Noise (\mathcal{N}): If ($v_j > v_{\min}$), the speed of the j th vehicle is reduced randomly by one with probability R , i.e.:

$$v'_j \leftarrow \max(v_{\min}, v_j - 1), \quad (2)$$

3. Proximity (\mathcal{P}) (to avoid collisions with other vehicles): The j th vehicle decelerates if it is in danger of colliding with the vehicle $j + 1$ in front of it at time $t + 1$. This represents *anticipatory driving*:

$$v'_j \leftarrow \min(v_j, d_j + v_{j+1}) \quad (3)$$

4. Movement (\mathcal{M}): Each vehicle is moved forward so that

$$x'_j = x_j + v_j \quad (4)$$

The principal modification that we make is thus in step \mathcal{D} ; in our model, each car has *prior knowledge* of the velocities of cars ahead of it, leading to a smoother, faster flow of traffic than that which is simply based on a rule such as [11], [18] $v_j = \min(v_j, d_j)$.

Step \mathcal{A} reflects the tendency of drivers to drive as fast as possible, without exceeding the maximum speed limit. The noise in the step \mathcal{R} takes into account stochastic braking to do with either road obstacles or individual reaction times of drivers; such obstacles, extrinsic or intrinsic, often result in the spontaneous formation of a traffic jam. The step \mathcal{P} implies that the driver of a car anticipates the position of the car in front of it at the next time step; if a collision looks likely, the driver brakes, but not otherwise. In other words, the driver would like to be at the maximal possible velocity consistent with the avoidance of collisions, as in automated traffic[3].

We apply the rules in the order $\mathcal{NPM A}$. Our initial investigations indicated that the order \mathcal{PNMA} led to several unphysical configurations, whereas $\mathcal{NPM A}$ did not. The reason for this is that with the noise being applied *after* the proximity step, cars were unable to adjust to the noise-reduced velocities of the traffic in front. This frequently led to an *artificial* jam, arising from the order of the rules rather than from the real dynamics of the system. Also, importantly, our choice of rules could be said to model the behavior of *anticipatory* drivers rather than, as in the case of the \mathcal{PNMA} ordering, *reactive* drivers. This choice reflects our wish to model automated cars[3].

3 The steady state in an open system: results and analysis

In this section, we describe both qualitative and quantitative features of our results for the steady state of traffic in an open system, as described by the

model presented above. We have performed extensive simulations of this, starting with different random initial conditions, different values for initial density ρ_{ini} , initial average velocity $\langle v_{ini} \rangle$ and the braking parameter R . Specifically, we have updated individual car velocities and positions in accord with the rules above. Regarding the boundaries, at each timestep, if the first position x_1 is empty, a vehicle is introduced with the maximum speed; correspondingly, vehicles travelling beyond x_L , are removed. After a transient period that depends on L , R , and on the (random) initial conditions, the system reaches its asymptotic steady state. We present in this paper only data obtained for system sizes $L = 400, 1000$ and $10,000$.

3.1 Results

In this subsection, we call attention to the three fundamental aspects of our results, which we will discuss in greater detail in the following. The first deals with the difference between the consequence of our rules and the NaSch rules in a *closed* system. The second concerns the collapse of the fundamental diagram, which is a *triangle* in the *closed* system[18], to a *point*, in an *open* system. *In other words, the nature of traffic flow on average is dominated not by the mean flux, but by the mean velocity or density - this is specific to traffic flow in open systems.* The third is really the central result of this paper. We find that the state of traffic in our system is determined by a) an extrinsic parameter, which turns out to be R and b) an intrinsic parameter, related to the density $\langle \rho \rangle = N/L$. This is analogous to models of vibrated granular media[25]; we suggest the use of two temperatures to describe fully the flow characteristics, and present initial evidence of this in terms of velocity correlation functions.

3.2 Open and closed systems: spacetime diagrams

Fig. 1 shows the difference engendered by our specific choice of rules with respect to the canonical NaSch model, in a *closed* system. We plot flux q vs. density ρ for a system of length $L = 10000$, and for a braking parameter $R = 0.4$. Our findings are:

- that the maximal value, q_{\max} of flux reached is enhanced when our specific rules are used. This is intuitively reasonable, since anticipatory driving would encourage the efficient throughput of cars on a highway.
- The highway capacity, which is defined as the density ρ_{\max} at which q_{\max} is attained, is *also* increased in our model of automated vehicles, with respect to the NaSch model. This too is in accord with intuition, and is a pointer to the advantage of the use of automated vehicles in practice.

In the following, we compare first the spacetime diagrams for open and closed systems with our rules, and next, spacetime diagrams corresponding to different regions of parameter space in an open system with our rules.

Fig. 2 is the spacetime diagram for a system with periodic boundary conditions, with $R = 0.7$, $\rho_{ini} = 0.2$ and $L = 400$, whereas Fig. 3 is the corresponding diagram for the same parameter set in an open system. We see that the closed system preserves its initial density, since cars cannot be 'lost' in a system with closed boundaries, but in the case of the open system, there is a transition from an initially low-density configuration to a jammed steady state. Thus, the open system 'chooses' its own final density, while the closed system simply maintains its initially chosen one. This is reinforced by Fig. 4, which is a spacetime diagram for an open system with $R = 0.5$, $\rho_{ini} = 0.7$ and $L = 400$. Here, in contrast to Fig. 3, an initially high-density system undergoes a transition to free flow in the steady-state, when it is characterised by a much lower value of the final density; we observe successive stages in the temporal evolution of traffic, with the formation, coarsening and eventual dissolution of jammed clusters.

We emphasise that in Figs. 3 and 4, the flux is a constant in the steady state; the rules of our model are such that strictly one car is introduced at the open left-hand end of the system (x_1), and, once the steady state is attained, one car leaves the right-hand end (x_L) of the system. Despite this, the nature of the flow is totally different in Figs. 3 and 4, the former indicating a transition from a free to a jammed state, and the latter the reverse. This already demonstrates that for an open system, the net flux is not a good descriptor of the traffic, an issue which we will address in greater detail in the next subsection.

3.3 The open system: flux, transients and the fundamental diagram

It is well known that for systems with periodic boundary conditions, the fundamental diagram of traffic flow is a triangle [2]. For an open system, however, this collapses to a point, in the sense that the average density $\langle \rho \rangle$ is always inversely proportional to the average velocity $\langle v \rangle$ *in the steady state*. Our reasoning is as follows. In our specific system, q_{in} , the incoming flux is strictly 1, while the outgoing flux q_{out} , in the steady state, will be constrained to be 1, thus implying an inverse relationship of velocity to density (since the net flux is a constant). However, even if the introduction of cars at the open left-hand end were to be more random, our argument would hold *on average*; in this case, whatever the average value $\langle q_{in} \rangle$ of the incoming flux, the average value $\langle q_{out} \rangle$ of the outgoing flux would be constrained to be the same in the steady state. Thus, once again, *in the steady state* we would see that the average density $\langle \rho \rangle$ would be inversely proportional to the average velocity $\langle v \rangle$. Fig. 5 illustrates this relationship, for a range of values of the braking parameter R . Note that the full line joining the points is meant as a guide to the eye, since in the intermediate regions, the system exhibits 'glassy' behaviour at least for finite system sizes (see below); this makes it very time-consuming to get systematic data within computationally reasonable times. However, our argument above is independent of such finite size effects, and is perfectly general.

All the above statements were made for the steady-state behaviour of our system. However, we have also [17] carried out detailed investigations of *tran-*

sient states in the open system, some examples of which are shown in Fig. 6. These figures are plots of the flux q versus time t , for different values of the braking parameter R ; in each plot, the different curves correspond to specific initial average velocities $\langle v_{ini} \rangle$, as indicated.

- We note first of all, that each of these systems asymptotes to the steady state where the flux is one (see above).
- A detailed investigation of all the simulations corresponding to the various curves in the figures allows us to make the following non-trivial observation: *all the curves which show an initial peak (i.e. where the value of the flux is greater than 1) are precisely those where the final steady state corresponds to free flow, whereas all curves which show an initial dip with a value of flux less than 1, are those where the final steady state corresponds to jammed traffic.*
- A combination of the above two arguments indicates strongly that flux is not a good descriptor of traffic in an open system, and that one has to deal with other descriptors (for example, average densities or velocities) to characterise the steady state of the system.
- We note that, as might be expected, increasing R leads to far fewer cases of free flow in the steady state.
- Finally we note that, for a given value of R , it is only the highest initial (average) velocities that manage to make the transition to free traffic. This shows already a strong hysteresis in the system. A simple picture to describe this situation is the following: for each value of the initial velocity indicated, the system is started with a fixed initial density. The peaks (and dips) are indicators of collective events, where the cars at a certain time cluster around selected values of velocities and densities, *during* the transient state; for example, a peak would indicate a 'rush' of cars, which eventually could 'un-jam' the system from an initially jammed state, in order eventually to make the transition to free flow. Conversely, all the dips correspond to the onset of a collective congestion, from which the system never recovers. We shall return to this point in the last section, but we already see indications of the strong influence of transients on steady-state traffic. This pinpoints the need to have an appropriate descriptor mirroring the initial conditions, in addition to the braking parameter R , in order to predict the steady-state behaviour of our traffic system.

In conclusion, we summarise the two main results of this section. The first is, that contrary to current ideas, flux cannot be used to characterise the state of traffic in an open system; high/low values of flux are commonly used to denote free/jammed traffic[2], but we have shown here that for the *same* values of flux, we can have either jammed or free traffic in an open system. The second result concerns the very interesting light that transient behaviour can throw on the steady-state behaviour of our traffic model (Fig. 6); this strong hysteretic

behaviour is reminiscent of granular flow, where the dynamics that take place during the formation of a granular bed determine its subsequent structure, as well as any resulting dynamics[23]. In the next section, we will further explore these analogies.

3.4 The nature of the transition as a function of ρ and R : analogies with granular flow

The behaviour of traffic transients presented in Fig. 6 showed that the initial velocity was a strong determinant of the steady state of the traffic in the sense that collective behaviour manifested either as 'bursts' or as 'blocks' for a significant, if transient, amount of time was able to free or jam the traffic. In Figures 7a and 7b, we explore the role of initial densities in this regard. The average behavior of the system is determined over an ensemble of different initial conditions: in particular, our results are averaged on an ensemble of 100 simulations for the *same* initial density and value of R . Fig. 7a is a plot of density ρ vs. time t (on a logarithmic scale) for $L = 400$, $\rho_{ini} = 0.7$, for a range of values of R , while Fig. 7b is the same plot for $\rho_{ini} = 0.2$. Our finding, in the high-density case represented by Fig. 7a, is that there is a strict phase separation between free and jammed traffic, with no intermediate states permissible. On the contrary, in Fig. 7b, we find a spread of final states. Even at the jammed and free ends, there is a spread of densities; for example, in the jammed phase, the range of steady-state densities achieved by our system ranges from ~ 0.85 to ~ 0.95 . However, and more interestingly, there is an intermediate region which does not reach the steady state even at arbitrarily long simulation times. In particular the data around $R = 0.51$ suggests that the density grows approximately logarithmically with time, an indication of 'glassy' behaviour at least for this system size.

To see the extent of finite size effects in this system, we performed checks for $L = 400, 1000$ and 10000 , with $\rho_{ini} = 0.2$ as well as, for comparison, the data for $L = 400$, with $\rho_{ini} = 0.7$. Our results are shown in Fig. 8, and suggest that the transition between free and jammed flow for $\rho_{ini} = 0.2$ gets sharper for larger systems. It is rather likely that in the limit of an infinitely large system, the 'glassy' states causing the spread in this transition for low initial densities might disappear and the curve might end up coinciding with that for high initial densities (see Fig. 8). However, it should be borne in mind that real highways are in fact of finite size, so that in our view the glassy states causing the difference between the low and high ρ_{ini} regimes are not just of physical, but possibly also of real interest.

We replot, in Fig. 9, our earlier data in terms of $\frac{d\rho}{dR}$ vs. R to show more clearly the nature of this transition for different initial densities; Fig. 9a corresponds to $L = 400, 10000$, with $\rho_{ini} = 0.2$, while Fig. 9b is same plot for a system size of $L = 400$, but with $\rho_{ini} = 0.7$. We note that, for the *same* system size, the transition from free to jammed traffic as a function of R is smeared out in the lower (initial) density case relative to the higher (initial) density one; the reason for this is that there will always be low-density configurations, even at

the highest values of R , which will 'escape' the jammed state and result in free traffic. (Fig. 10 illustrates this situation for $L = 400$, $R = 0.8$, with $\rho_{ini} = 0.2$. Each of the lines in the figure corresponds to a run with these parameters, and we note that while most of them end up in a jammed state, there are two runs which result in free traffic). We however, also note that the width of the transition in the low initial density case is greatly reduced for larger systems, although the qualitative features appear to survive, e.g. the cusp in the curve. It appears to us from these and other observations, that there will always be a difference in the nature of the transition for the cases when arbitrarily large systems of the *same* size are started at low and high densities (i.e. above and below about 0.5).

What this makes clear is that contrary to some earlier speculations [26], traffic in finite systems cannot be characterised by a single 'temperature'-like variable; thus for example, in our case, it is not enough just to regard R as an effective temperature, which determines the state that the system will reach at asymptotically long times. This underlines the need to consider both the density ρ and the braking parameter R when one tries to predict the asymptotic state of traffic in open systems.

The results of Figs. 8 and 9 suggest a clearly hysteretic behaviour of the system, which we interpret as follows. In real traffic, it is well known that the variability of road and weather conditions even on a single highway would lead to a variability in braking rates. With this in mind, we imagine starting in a jammed configuration (of density $\rho_{ini} = 0.7$, say as in Fig. 9b), on a road where the effective braking parameter has a value of 0.8. Suddenly the road conditions change dramatically, and the effective braking parameter becomes 0.2, say. We see from Fig. 8, that the resultant value of the density of traffic would end up being around 0.2, say. Now let us imagine a sudden return to bad weather/road conditions, resulting in an effective braking parameter of $= 0.8$, as before. What Figure 9a as well as Fig. 10 indicate is that one can either remain in a state of free traffic, or end up in a jammed state, depending clearly on the precise positional correlations of the cars; in other words, there is a *finite probability that one will not return to the initial state*, i.e. in this case, the one that was characterised by a density of 0.7. This indicates that if we were to plot a curve of ρ vs. R , with these variability conditions incorporated, we would expect to see a hysteresis loop in density ρ , the size of which would clearly depend on the length of road L for which each value of R would need to be implemented. Current work is in progress to confirm this.

This behaviour is very reminiscent of that which obtains in granular media. Imagine a box of sand is subjected to 'annealed cooling' [27]; that is, it is submitted to different shaking intensities Γ for variable amounts of time $\Delta\tau$, such that after each time $\Delta\tau$, there is a jump in shaking intensity $\Delta\Gamma$ to the next shaking intensity. It is found experimentally [27] that a hysteresis loop is traced out in terms of density, the size of which depends [23, 16] on the specific ratio $\frac{\Delta\Gamma}{\Delta\tau}$, known as the 'ramp rate' of the system. For small ramp rates, i.e. where each value of Γ is traced out quasi-continuously, and where the system is allowed to 'equilibrate' at each value of the shaking intensity, the hysteresis

loop is much smaller than at larger ramp rates. The same would obtain in our traffic flow system if we made the following analogies:

Granular Flow	Traffic Flow
vibration intensity Γ	inverse braking parameter $\frac{1}{R}$
waiting time $\Delta\tau$	'effective system size' L
density ρ	density ρ

Based on this, we make a further analogy to do with effective temperatures in the case of traffic flow. In the case of granular media, it is now conventional to refer to the vibration intensity and density respectively as being related to effective temperatures corresponding to the fast and slow dynamics of this athermal and complex system; a related fast dynamics 'temperature' has been in use for a long time by the engineering community [28], while a version of the slow dynamics temperature (termed the compactivity) was first proposed by Edwards and collaborators[21]. Subsequent work has confirmed the need to use both [29, 25] these temperatures in any analysis of the dynamics of granular media. We here thus propose, based on our present results:

- the use of the inverse braking parameter $\frac{1}{R}$ as an effective temperature which controls the 'fast' or 'single-car' dynamics of traffic flow
- the use of the inverse density $\frac{1}{\rho}$ as an effective temperature which controls the 'slow' or 'collective' dynamics of traffic flow

Clearly, this analogy is relatively qualitative at this point, and in this paper, we quantify it to a slightly greater extent with the use of velocity correlation function. However, we emphasise that more analysis, especially to do with the tracing out of the hysteresis curve referred to earlier, is in progress.

In this tentative spirit, we examine equal-time velocity correlation functions as a function of space, in the steady state, for systems of traffic which are started out with rather different initial conditions. In Fig. 11a, we present averages over 200 runs on a system of size $L = 400$ for the quantity $\langle v(0, t)v(x, t) \rangle$ vs space x with an initial density of $\rho_{ini} = 0.7$ and different values of R , as designated on the legend of the figure. In Fig. 11b, corresponding data are presented for systems with an initial density of $\rho_{ini} = 0.2$ and $R = 0.3$, and 0.8.

- In Fig. 11a, we notice that for $R > 0.5$, there is little free volume for the cars to move, so that the initial jammed state persists more or less. At $R = 0.5$, there is the appearance of a shell structure, which becomes increasingly evident for progressively lower values of R . We note that in fact, for $R \in (0.4, 0.5)$, the minima and the maxima of the shells are the sharpest, with the most regularity in shell spacing; for lower values, the structure becomes more diffuse, until at $R = 0.2$, we have a 'liquid-like'

structure. This might be an indicator that the jammed state gives way to an 'ordered' state for $R \sim 0.4$, and the ordering disappears rapidly as we reach lower values of the braking parameter. This is analogous to the situation in granular media: starting from totally jammed configurations (typically characterised by a value of the density corresponding to random close packing [30], although this has recently been the subject of considerable debate[31]), when the grains have just enough free volume to move, it is known that the system moves preferentially to configurations which have some semblance of order [23, 25]. As the excitation intensity Γ increases, this order gives way to a liquid-like structure, characterised by more diffuse spatiotemporal correlation functions.

- In Fig. 11b, we notice a rather distinctive difference. As remarked before, the low-density system, even at high values of the braking parameter R , has a small but finite probability to remain in the free traffic phase. Thus a very small fraction of the runs carried out at $R = 0.8$ result in the free state (cf. Fig. 10), while typically the system ends up in the congested state; it is therefore rather difficult to say anything conclusive about the free state at $R = 0.8$, but we can see rather clearly that the jammed state in this case shows some structure, which is somewhat different to that manifested by the jammed state in Fig. 11a. This is another manifestation of strong hysteretic effects, referred to earlier. We speculate a particular reason for the appearance of some shell-like structure in this case (as contrasted with the totally jammed corresponding configuration in Fig. 11a) may be the formation of the 'free zone' manifested in Fig. 3 at the open end, which appears to be rather typical[17] of jammed states reached from initially low-density configurations. When $R = 0.3$, the structure of the system is entirely 'liquid-like', and not distinguishable from the liquid-like state reached in Fig. 11a (when the system was started at a high initial density). This liquid-like structure in both cases is reminiscent of the fluidised state in a granular medium shaken at high intensities of vibration [23, 16].

Additionally, we remark on the specific meaning of such *dynamical* correlations; in analogy with earlier work on granular flow [23], we define a *dynamical cluster* for a given R as being the number of sites which are within the first shell of the velocity correlation function. *The physical import of a dynamical cluster is that it reflects the range over which cars are correlated in their velocities.* In general, as fewer cars face random obstacles, more and more of them develop velocity correlations, i.e. they begin to 'move together' in clumps. Returning to the analogy with granular flow, this mirrors the situation found in earlier work [23] where a *decrease in external perturbations applied to a granular system* (or in our case *an increase in the braking parameter R*) causes an *increase* in the size of a typical dynamical cluster of grains. We remark here that dynamical correlations of a similar sort have also subsequently been observed in glasses, where they are commonly referred to as spatial heterogeneities[32]. This similarity of behaviour of our traffic flow model with granular and glassy materials

is an additional indicator that two temperatures are necessary to characterise our system, since recent work in both glassy[33] and granular[25, 16] materials has indicated the ubiquity of this description in both systems. Further work is in progress to quantify these analogies, especially to do with unequal time correlation functions.

4 Discussion

In this paper we have analysed a model representing traffic flow in a system which involves automated cars, and hence anticipatory driving, which was motivated by current interest on automated highways in real traffic systems [3]. While we have here focused on the statistical mechanical aspects of this model, we point out that a very detailed discussion of the practical aspects of our model, with special relevance to real automated vehicles, can be found in[17, 34].

One of our major results concerns the difference between open and closed traffic systems. In particular, in the former, we find that the nature of the phase diagram is completely altered with respect to the latter; for example, the fundamental diagram of flux versus density as a function of the parameter R presented recently for closed systems [19, 2] collapses to a point in the case of an open system. We find thus that flux is not a good descriptor of open systems, and that a knowledge of densities and velocities is necessary to characterise the difference between free and jammed traffic. This is indicated both by the steady-state, as well as the transient behaviour of our traffic model.

Our other major result concerns the 'glassy' or first-order nature of the dynamics of our traffic flow model in an open system of finite size. We have shown, both by a detailed examination of the effect of varying the initial density of the traffic system, and then subjecting it to varying braking parameters R , as well as by an examination of equal-time correlation functions, that two effective temperatures are needed to characterise the steady state of our traffic model. As in the case of granular flow, another example of an athermal complex system which exhibits glassy dynamics, we suggest that the 'fast' (single-particle) dynamics temperature is related to the external perturbation (vibration intensity in the case of granular media, and the inverse braking parameter $\frac{1}{R}$ in our case), while the 'slow' (collective) dynamics temperature is related in both cases to the inverse of the density ρ .

Further work is in progress to examine this analogy in greater detail, in particular to do with aspects related to the hysteresis curve referred to above, as well as the analysis of two-time correlation functions in our present model.

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6 Figure Captions

Figure 1. Fundamental diagrams for the closed system comparing our model with the NaSch model, for $R = 0.4$ and $L = 10,000$. Note that the maximal flux as well as the highway capacity are enhanced in our model with respect to NaSch.

Figure 2. Spacetime diagram for traffic flow in a closed system ($L = 400$) corresponding to a braking probability $R = 0.7$, and starting with an initial density $\rho_{ini} = 0.2$ ($v = 1$ in red, $v = 2$ in orange, $v = 3$ in yellow, $v = 4$ in green and $v = 5$ in blue). The system preserves its initial density.

Figure 3. Spacetime diagram for traffic flow in an open system ($L = 400$) corresponding to a braking probability $R = 0.7$, and a density $\rho = 0.2$ ($v = 1$ in red, $v = 2$ in orange, $v = 3$ in yellow, $v = 4$ in green and $v = 5$ in blue). An initial state of free traffic gives way to a congested state. Note, however, the persistence of a small zone of free traffic near the left-hand boundary (see section 3).

Figure 4. Spacetime diagram for traffic flow in an open system ($L = 400$) corresponding to a braking probability $R = 0.2$, and a density $\rho = 0.7$. An initially jammed state gives way to freely flowing traffic. Note the formation, coarsening, and eventual dissolution of areas of congestion before the steady state is reached.

Figure 5. Plots of the average density $\langle \rho \rangle$ vs. the average velocity $\langle v \rangle$ in the steady state, with different initial conditions, and $L = 400$, for different values of the braking parameter R values indicated by different symbols on the figure. Note that the solid line is meant as a guide to the eye.

Figure 6. Plots of the time evolution of the density of traffic in an open system for an initial density $\langle \rho_{ini} \rangle = 0.4$ and $L = 400$. The different figures correspond to different values of the braking parameter R as indicated by different symbols on the figure. On each figure, the differently coloured lines correspond to different values of $\langle v_{ini} \rangle$, also indicated on the respective figures. Note that all curves which have an initial peak result in a steady state of free traffic, whereas the reverse is the case for all curves which manifest an initial dip, thus emphasising the importance of transient behaviour for the steady state of traffic.

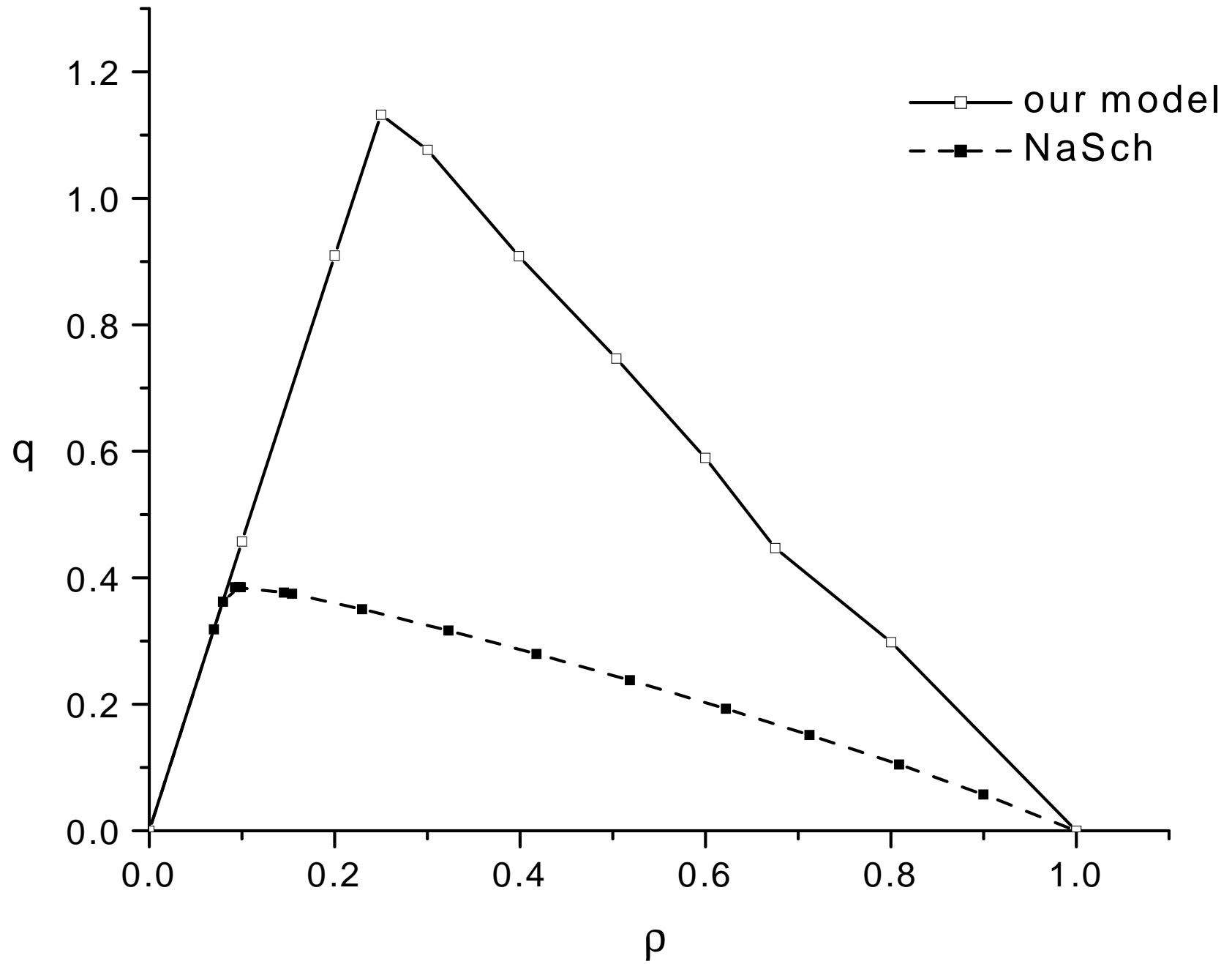
Figure 7. Plots of density ρ vs. time t (on a logarithmic scale) for $L = 400$, and a) $\rho_{ini} = 0.7$, and b) $\rho_{ini} = 0.2$ for a range of values of R . In Fig. 7a, note the strict phase separation between free and jammed traffic, with no intermediate states. In Fig. 7b, there is a spread of final states. In particular the data around $R = 0.51$ suggests that the density grows approximately logarithmically with time, an indication of 'glassy' behaviour at least for this system size.

Figure 8. Plots of density ρ vs. braking parameter R for system sizes $L = 400, 1000$ and 10000 , with $\rho_{ini} = 0.2$. The data for $\rho_{ini} = 0.7$ with $L = 400$, are shown for comparison.

Figure 9. Plots of $\frac{d\rho}{dR}$ vs R for a) $\rho_{ini} = 0.2$ and $L = 400, 10000$ b) for $\rho_{ini} = 0.7$ and $L = 400$. Note the qualitative similarity between the curves in a), despite their difference in size.

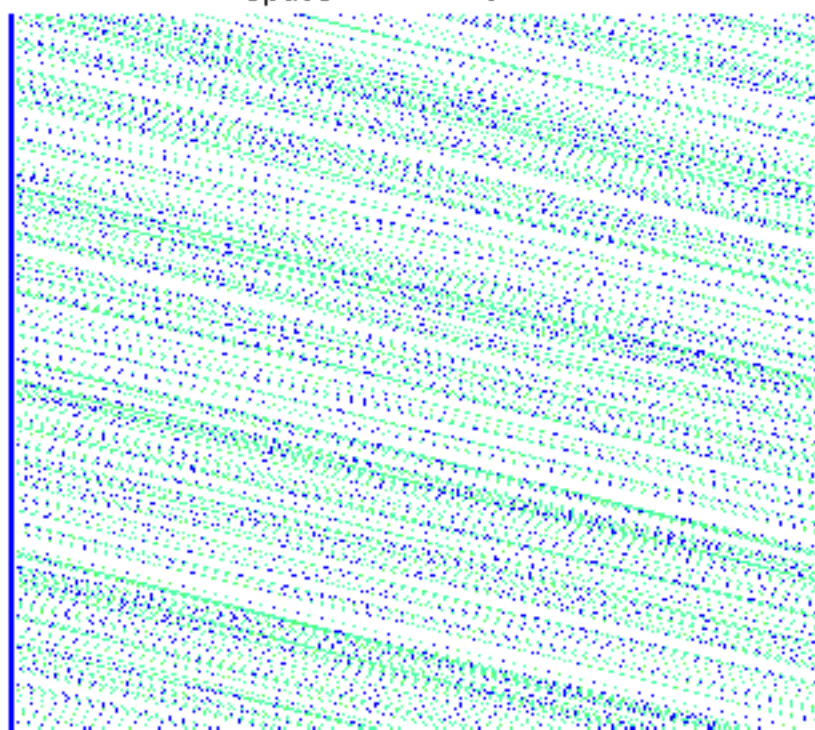
Figure 10. Plots of density ρ vs. time t for $L = 400$, and $\rho_{ini} = 0.2$. The lines correspond to different runs with the *same* initial densities but *different positional correlations* between the cars, which result in a spread of different steady states.

Figure 11. Plots of velocity-velocity correlation functions $\langle v_x v_{x'} \rangle$ for a) $\rho_{ini} = 0.7$ b) $\rho_{ini} = 0.2$ for $L = 400$, corresponding to a range of different values of the braking probability R , whose values are indicated by the legends on the plots. In a), note the collapse of the curves for all values of $R \geq 0.6$. Note that the 'shells' for lower values of R get increasingly diffuse, until the liquid-like structure corresponding to $R = 0.2$ is reached. In b) note the difference of the jammed state (red line) with the corresponding jammed state (yellow line) in a), which confirms the hysteretic behaviour. The free state reached for $R = 0.3$, is, however virtually identical to the free state reached in a), indicating a 'liquid-like' structure.

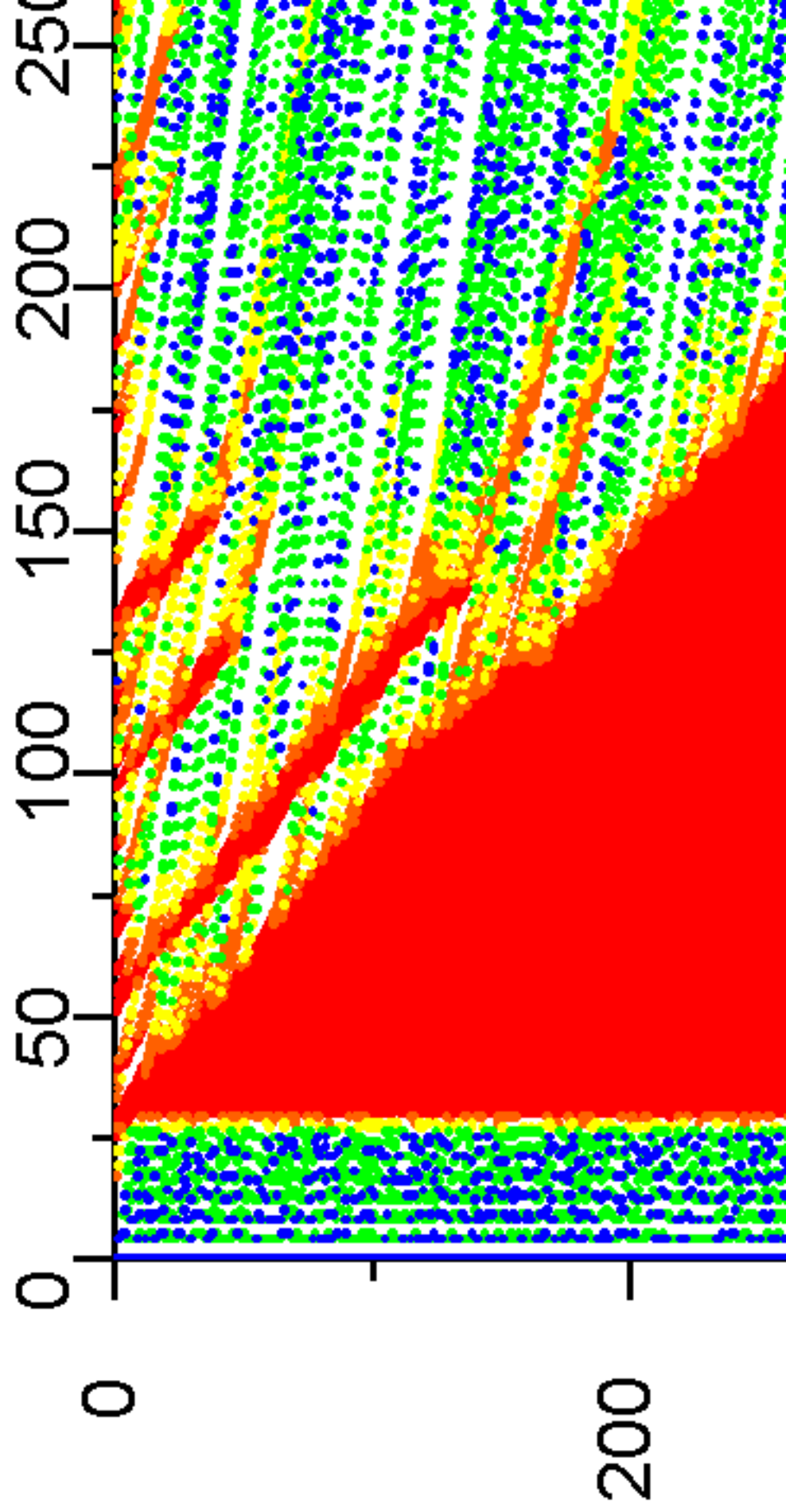


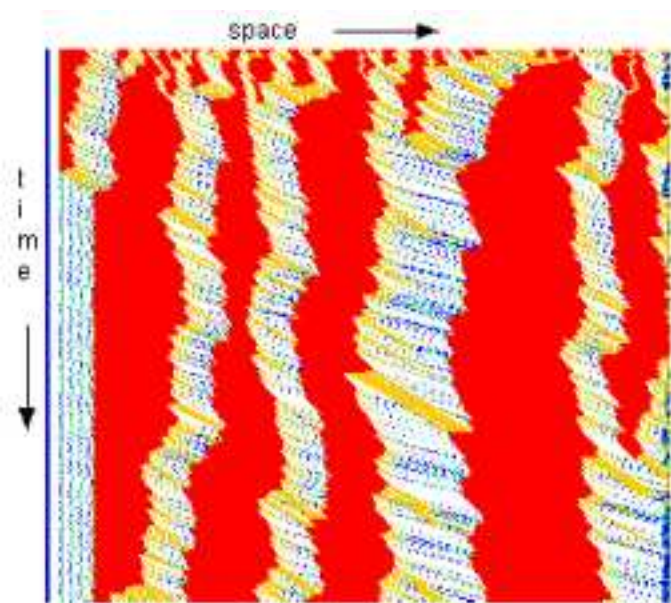
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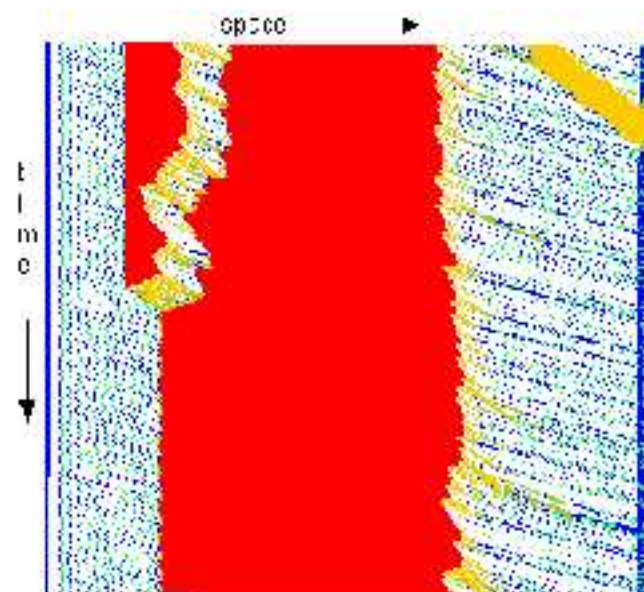


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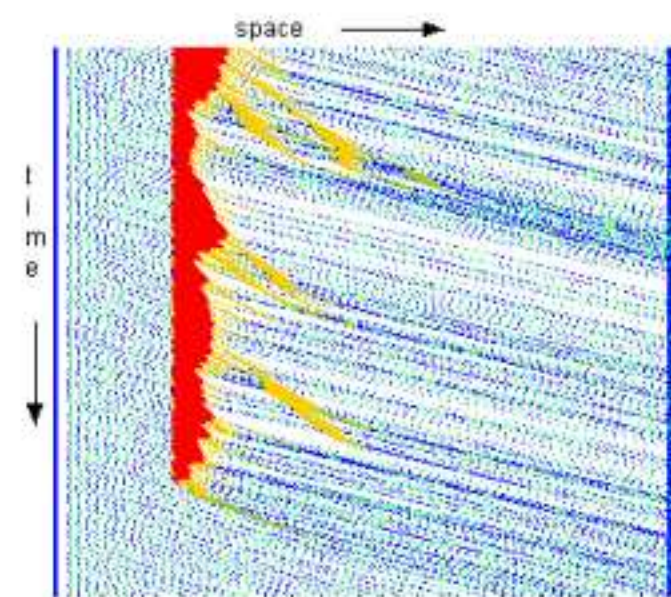




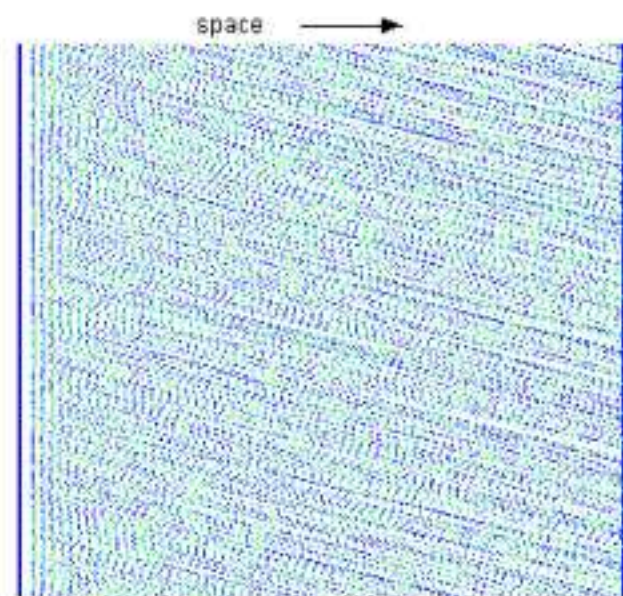
a)



b)



c)



d)

